SELECTIVE TRANSPORT AND ARMOURED LAYER DEVELOPMENT IN NON-UNIFORM BED MATERIALS PART 1: NUMERICAL MODEL DEVELOPMENT

Arie Setiadi Moerwanto

Pusat Penelitian dan Pengembangan Sumber Daya Air
Jl. H. Juanda no. 193, Bandung
E-Mail: ariemoerwanto@yahoo.com

Diterima: 30 Maret 2011; Disetujui: 10 Agustus 2011

ABSTRAK

Pada sungai dengan material dasar sungai tidak seragam, ukuran butir, rapat massa dan keberadaan butir-butir material terhadap aliran sungai akan sangat mempengaruhi gerak mula masing-masing butir material dasar sungai. Aliran sungai akan terlebih dahulu mengangkut material dasar sungai dengan ukuran butir yang relatif kecil dan tidak terlindungi, sementara butir-butir dengan ukuran yang lebih besar dan stabil akan membentuk lapisan perisai di permukaan dasar sungai. Karena angkutan sedimen terpilah, pembentukan lapisan perisai dan pembentukan corak dasar sungai sangat terkait erat dengan laju angkutan sedimen, maka kecermatan penyimulasian fenomena-fenomena tersebut akan sangat mempengaruhi ketelitian analisis kedalaman aliran, laju muatan sedimen dasar, laju angkutan sedimen layang dan pada akhirnya juga akan mempengaruhi kecermatan analisis perubahan morfologi sungai. Dalam tulisan bagian pertama ini dibahas dengan detail studi pustaka dan landasan teori untuk pengembangan model numerik angkutan material sedimen terpilah, prediksi ketebalan lapisan aktif, mekanisme pertukaran antara material dasar sungai, angkutan sedimen dasar dan angkutan sedimen layang, pembentukan corak dasar sungai dan pembentukan lapisan perisai pada sungai dengan material dasar sungai tidak seragam. Verifikasi dan penilaian unjuk kerja model numerik yang didesain diulas dalam tulisan bagian kedua.

Kata Kunci: *Material tidak seragam dasar sungai, angkutan sedimen terpilah, corak dasar sungai, lapisan perisai, morfologi sungai.*

ABSTRACT

For rivers with non-uniform bed materials, particle size, shape, density and exposure to the flow determine the resistance to movement of an individual particle. River flow transports the exposed, finer particles first, while the larger and more stable particles tend to form clusters of particles, namely armoured layers on the bed surface. Since the selective transport, armoured layer development and geometry of bed forms are integrally linked to the bed material transport, hence the accuracy of simulating those phenomena will also influence the accuracy of analysis of flow depth, the bed load transport rate, the suspended load concentration and later on river morphology changes. This first part of two research papers describes in detail the literature review and theoretical background to support numerical model development of selective transport, prediction of active layer thickness, schematization of bed materials exchange, and defining alluvial bed roughness and armoured layer development in non-uniform bed materials. Verifying and testing of the developed model performance will be presented in the second part of this research paper.

Keywords: *Non-uniform bed materials, selective sediment transport, bed forms, armoured layer, river morphology.*

INTRODUCTION

The armouring process is important in river engineering studies, for example, the problem of river bed degradation downstream of a dam due to the entrapment of bed material in the reservoir and the release of sediment-free water. For a non-uniform bed material, particle size, shape, density and exposure to the flow determine the resistance to movement of an individual particle. Typically, the flow transports the exposed, finer particles first. At the same time, the larger and more stable particles tend to form clusters of particles on the bed surface. However, these clusters are not permanent. The flow undermines the edges, resulting in a break-up of the clusters, some of the particles are transported, and new clusters are formed. The armouring is an asymptotic process (Chin et al., 1994). The rate of sediment transport from the bed surface decreases with time.

Thoroughly elaboration on theoretical and practical aspect of non-uniform sediment transport under unsteady flow conditions was given by Moerwanto (2011). While, extensive flume experiments were carried out at the Department of Civil Engineering, University of Auckland to gain a detailed understanding of the development of armoured layers in non-uniform bed materials (Chin, 1985 and Chin et al., 1994).

AIMS

This study is aimed to develop a sub-numerical model that able to simulate selective transport and armoured layer development in non-uniform bed materials under unsteady flow conditions using finite element method. This study is carried out under the scheme of developing A Riverine Fully Coupled Finite Element Model with Sediment Transport Sub-processes as a required tool to support the adaptation policy to respond to the global climate change and been setup by considering the following aspects:

1 the adoption of the dynamic wave approach by retaining all the terms in the momentum equation,
2 the explicit separation of the bed load and the suspended load transport,
3 the vertical exchange of sediment and armored layer development and a means of handling non-uniform distribution of bed material, and
4 incorporating the spatial and temporal lag effects of bed load transport as a consequence of the dynamic wave approach.

However, this first part of two research papers will only describe in detail the literature review and theoretical background to support numerical model development.

LITERATURE REVIEW

A channel bed material is normally classified as non-uniform whenever its geometric standard deviation value $\sigma_g = \frac{D_{84}}{D_{16}} \geq 5$ (Ackers and White, 1980). In the case of a channel with non-uniform bed material, two distinct phenomena concerning the grain mobility appear (Einstein, 1950; Egiazaroff, 1965 and Day, 1980). The mobility of particles with a grain size D_j in a uniform bed consisting entirely of these particles can be compared with the mobility of those particles in a non-uniform bed with a mean particle size of the mixture D_m.

1 For D_j > D_m, the mobility of the particles of size D_j is higher when they are part of a mixture of grain sizes than if the bed had a uniform grain size D_j.
2 For D_j < D_m, the mobility of the particles of size D_j is less when they are part of a mixture of grain sizes than if the bed had a uniform grain size D_j.

where

D_m , $\sum_{j=1}^{J_{frac}} f_j . D_j$, [m],

D_j , representative diameter of the size class j, [m];

f_j , volume fraction of sediment size class j being present in the active layer, [-];

J_{frac} , total number of discrete sediment size classes used to represent the non-uniform bed material, [-];

j , size class counter.

In other words, the larger grains have an acceleration effect on the movement of the mixture. Meanwhile, the larger grains also tend to shelter the smaller grains. The first phenomenon is known as the exposure effect, while the latter is known as the shielding effect.

To deal with selective transport and armoured layer development in non-uniform bed material, the following phenomena must be properly managed: formulation of size fraction sediment transport, prediction of active layer thickness, schematization of bed materials exchange, and defining alluvial bed roughness.

1 Formulation of Size Fraction Transport

A comparison between theoretical results with field and prototype measurements has shown that non-uniform sediment cannot be effectively represented by a unique grain diameter (Egiazaroff, 1965). A fraction by fraction estimation is suggested to predict the bed load transport of a channel with non-uniform bed material. For this purpose, an existing formula for uniform bed material is modified to handle non-uniform bed material. An exposure correction factor is normally adopted to account for the exposure and shielding effects. The exposure correction factor is defined as a function of grain size and is in the form of a ratio involving the controlling variables of a particular transport formula. The exposure correction factor is required to force the transport formula to give the same transport rate for each size class of the graded material as was obtained in the experiments (Proffitt and Sutherland, 1983). Two types of correction can be distinguished:

1) Correction of the effective or grain shear stress, $\tau_b^{'}$, i.e. its value for the finer size

classes is decreased to include the shielding effect, while increasing its value for coarser size classes to accommodate the exposure effect (Einstein, 1950; Day, 1980; Proffitt and Sutherland, 1983).

2) Correction of the critical bed shear stress, τ_{cr}, i.e. its value for the finer size classes is increased to account for the shielding effect, while decreasing its value for the coarser size classes to accommodate the exposure effect (Egiazaroff, 1965; Ashida and Michiue, 1973).

Where:

$\tau_b' = \mu \rho g h I_f$, grain shear stress, [N/m^2];

$\mu = \dfrac{C^2}{(C')^2}$, ripple factor, [-];

$C' = 18 \log \dfrac{12h}{D_{90}}$, grain-related Chezy coefficient for bed roughness, [m$^{0.5}$/s].

τ_{cr}, critical bed shear stress according to Shields, [N/m^2].

The general formula for bed load transport of a non-uniform sediment can be written as:

$$Q_{sb} = \sum_{j=1}^{J_{frac}} f_j \times \mathrm{f}\left(\xi_j, D_j, h, \tau_b', \tau_{cr}\right) \tag{1}$$

where

Q_{sb}, bed load transport capacity, [m^3/s];

ξ_j, exposure correction factor of sediment size class *j*, [-];

$\mathrm{f}\left(\xi_j, D_j, h, \tau_b', \tau_{cr}\right)$, bed load transport capacity of the size class *j* for the case of uniform bed material in the identical hydraulic conditions, [m^3/s].

1) Exposure correction factors for the grain shear stress

An example of the exposure correction factor that can be categorised as the grain shear stress correction type was described by Einstein (1950). To accommodate the exposure and shielding effects, Einstein proposed the following exposure correction factor for the grain shear stress:

$$\tau'_{b_j(corrected)} = \frac{\tau'_{b_j}}{\xi_j}$$

where

τ'_{b_j}, grain shear stress for sediment size class *j*, [N/m^2];

$\tau'_{b_j(corrected)}$, corrected value of τ_b' for sediment size class *j*, [N/m^2].

It was proposed that for the coarser size classes, ξ_j = 1, while for the finer size classes, $\xi_j > 1$ (see Figure 1). Many of the bed load sediment transport capacity formulas can normally be transformed into the form of $Q_{sb_j} = \mathrm{f}\left(\tau'_{b_j} - \tau_{cr_j}\right)$. When the corrected grain shear stress is substituted, the following relation can be derived:

$$Q_{sb_j} = \mathrm{f}\left(\tau'_{b_j(corrected)} - \tau_{cr_j}\right)$$

$$Q_{sb_j} = \mathrm{f}\left(\frac{\tau'_{b_j}}{\xi_j} - \tau_{cr_j}\right)$$

It can be seen that Einstein's correction factor results in a reduction in the bed load transport rate for the finer size classes.

Other types of exposure correction factor for the grain shear stress were indirectly introduced by Proffitt and Sutherland (1983) and Day (1980).

The various correction factors for the grain shear stress are compared in Figure 1. Even though each correction factor was derived from a different starting point, it is evident from Figure 1 that the trends are similar in representing the exposure and shielding phenomena. A distinctive, in terms of trend and magnitude, exposure correction factor is given by Einstein's relation. Einstein's relation gives a shielding correction for finer size classes up to an order of magnitude higher than the other formulations. On the contrary, no exposure correction is given for the coarser size classes. The discrepancy of Einstein's correction factor has also been commented by Gessler (1967).

2) Exposure correction factors for the critical bed shear stress

A simple approach to estimate the required exposure correction factor for the critical bed shear stress was given by Ribberink (1987) based on the following Meyer-Peter and Müller (MPM) bed load transport formula:

$$Q_{sb_j} = 13.3 f_j \left(\Delta g D_j^3\right)^{0.5} \left(\tau^*_{b_j} - \tau^*_{cr}\right)^{1.5} B_{act} \tag{2}$$

where

$\tau^{*'}_{b_j} = \dfrac{\mu u_*^2}{\Delta g D_j}$, dimensionless grain shear stress for the size class *j*;

$\mu = \dfrac{\tau_b'}{\tau_b}$, ripple factor, [-];

$\tau^*_{cr} = \dfrac{\tau_{cr}}{(\rho_s - \rho) g D_j}$, dimensionless critical bed shear stress.

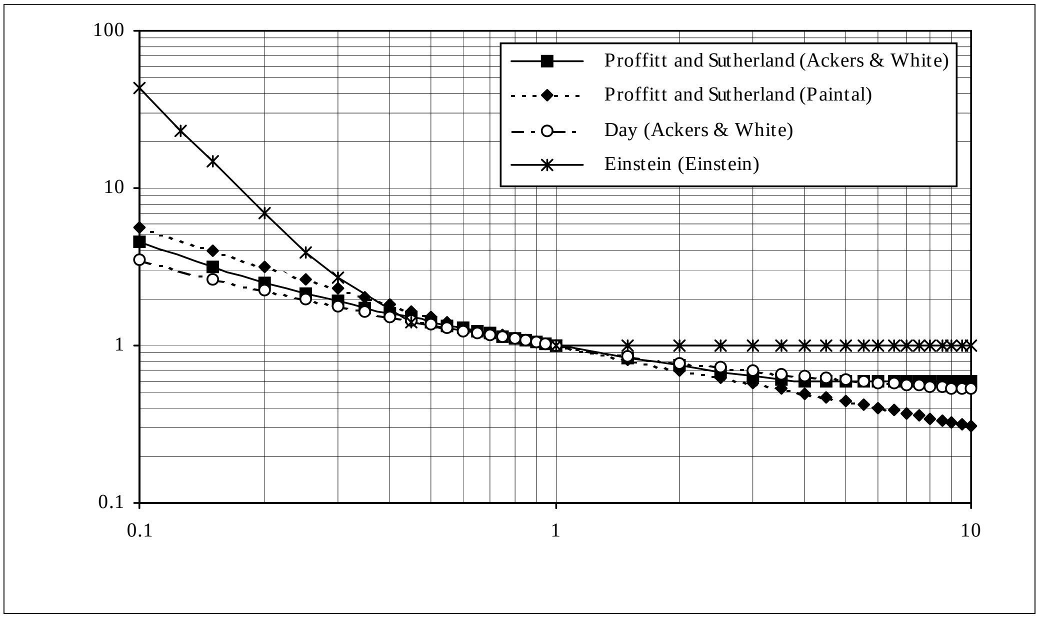

Figure 1 Fractional exposure correction factor of the grain shear stress for a non-uniform bed material (after Ribberink, 1987).

The pivotal assumption of the Ribberink exposure correction factor is that the distribution of the transported sediment is equal to the distribution of material in the active layer. Based on this assumption, the following relation can be derived:

$$f_{T_j} = \frac{Q_{sb_j}}{Q_{sb}} = f_j \text{ , or } \frac{Q_{sb_j}}{f_j} = Q_{sb} \qquad \textbf{(4)}$$

where

f_{T_j} , volume fraction of sediment size class j being present in the transported sediment, [-].

Comparing Equation **(3)** with Equation **(4)** leads to a condition that the term $\left(\tau^*_{cr}.D_j\right)$ in Equation **(3)** needs to have the same value for all size classes. A correction factor for the parameter τ^*_{cr} is needed. Assuming that no correction factor is needed for the sediment size class of which the representative diameter is equal to the average diameter of the bed material, D_m , the following exposure correction factor for the critical bed shear stress can be derived:

$$\xi_j\ \tau^*_{cr}\ D_j = 1.0\ \tau^*_{cr}\ D_m = \text{constant ,}$$

$$\text{hence } \xi_j = \frac{D_m}{D_j} \qquad \textbf{(5)}$$

Ribberink's exposure correction factors for the critical bed shear stress have been plotted in Figure 2. Other types of exposure correction factor for the critical bed shear stress were introduced by Egiazaroff (1965), and Ashida and Michiue (1973). A comparison of the various exposure correction factors for the critical bed shear stress is illustrated in Figure 2. It is evident from Figure 2 that the agreement between them is better than the agreement of the various exposure correction factors for the effective shear stress. The proposed modification by Ashida and Michiue for Egiazaroff's exposure correction factor only resulted in a slight change compared with that of the original correction factor. These additional data shows the reliability of Egiazaroff's exposure correction factor.

2 Active Layer Thickness

In the present study, the active layer is defined as the upper level of a stream bed in which continual mixing due to turbulence, bed form migration and the cumulative effect of selective transport and deposition are occuring (Bennett and Nordin, 1977). The bed material gradation of the active layer is assumed to be homogenous across the active width and also over the space step. The bed material contained in the active layer is used to represent the bed material of the channel. Therefore, at each time step of a simulation only this material is used:

a to characterise the bed roughness, to characterise the bed load sediment transport, and
b to be transported and sorted by the flowing water.

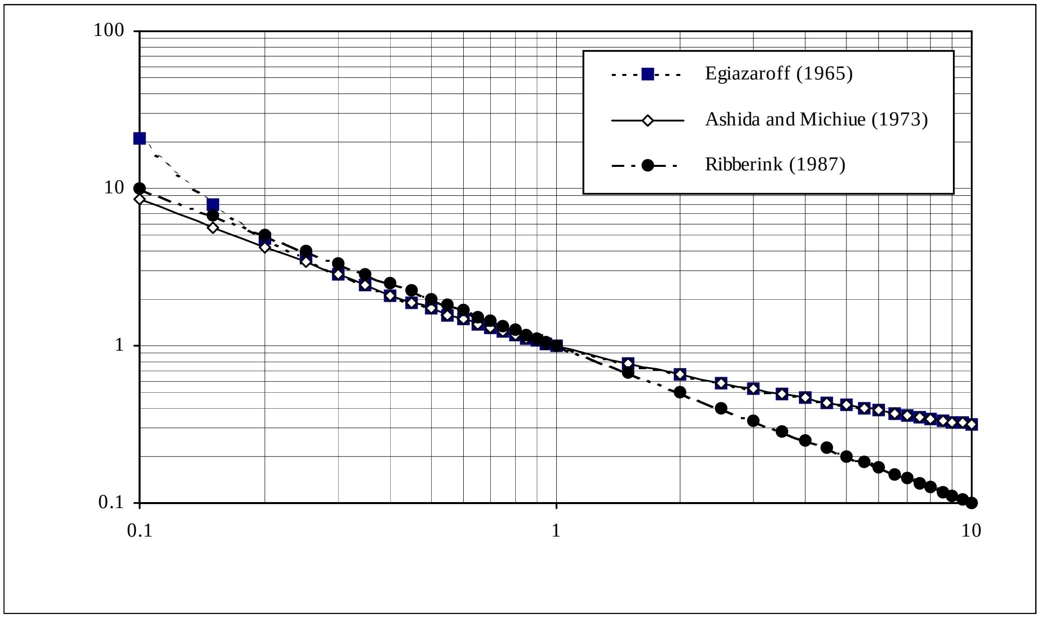

Figure 2 Fractional exposure correction factor for the critical bed shear stress of a non-uniform bed material (after Ribberink, 1987).

The distribution of the material contained in the active layer is evaluated and updated at each time step. The thickness of the active layer needs to be determined. The literature survey reveals that the thickness of the layer is normally related to:

a) the gradation of the bed material (Bennett and Nordin, 1977; Borah et al., 1982), or
b) the height of the bed forms and their rate of movement (Ribberink, 1987; Holly, 1988; Rahuel et al., 1989; Holly and Rahuel, 1990).

1) Thickness of the active layer based on the gradation of the bed material

Borah et al., (1982) defined the active layer thickness mainly by considering the particle size distribution, the porosity and the critical shear stress as follows:

$$\delta = \frac{D_{J_L}}{(1-p)\sum_{j=J_L}^{J_{frac}} f_j} \qquad \textbf{(6)}$$

where

δ , thickness of the active layer, [m];

D_{J_L} , representative grain diameter of the $J_L{}^{th}$ size class, [m];

J_L , the $J_L{}^{th}$ size class, in which its representative grain diameter D_{J_L} , is the smallest size of bed material that cannot be transported by the flow, [-];

J_{frac} , total number of discrete sediment size classes used to represent the non-uniform bed material.

p , porosity of bed material, [-].

The critical bed shear stress is used to determine the smallest representative grain diameter of the size class that cannot be transported by the flow. The summation in Equation **(6)** is in fact the summation over the size classes that are not transported by the flow.

Bennett and Nordin (1977) selected a constant thickness for the active layer throughout a simulation. The thickness of the active layer was taken to be N times of the largest representative diameter of the sediment size class used in a simulation. No detailed guidance was given to specify the parameter constant N. However, it was proposed to take into account the height of bed forms and their rate of movement. Several preliminary and calibration tests were required to achieve the appropriate value of N and to meet the simulated field conditions.

2) Thickness of the active layer based on the bed form height

Bennett and Nordin (1977), Ribberink (1987), and Holly (1988) determined the thickness of an active layer by considering the height of the bed forms as:

$$\delta = \beta . H \qquad \textbf{(7)}$$

where

β , shape factor, which is equal to 1/2, 2/3 or $2/\pi$, for triangular, parabolic or sinusoidal bed form shapes respectively, [-];

H , height of the bed forms, [m].

Most bed form predictors are proposed for dunes which are normally generated in the lower transport regime of channels with a moveable bed. The simplest dune height predictor was given by Allen (1965), which was based on field data.

$$H = 0.086h^{1.19} \quad \textbf{(8)}$$

where

h , flow depth from the water surface to the mean bed level (i.e. at half the bed forms height), [m].

This formula does not represent the development and breaking up phases of dunes. Field and flume data have shown that various bed forms may develop, (e.g. flat bed, ripples or dunes) at the same flow depth. This phenomenon leads to condition that adoption of this approach must carefully be taken.

Gill (1971) proposed a dune height predictor based on an analysis of the bed load transport rate and the migration velocity of the dunes. After combining the data with a transport formula of the type of $Q_{sb} = a\left(\tau_b^{*'} - \tau_{cr}^*\right)^b$, Gill (1971) proposed the following dune height predictor:

$$\frac{H}{h} = \frac{1}{2b\beta}\left(1 - \frac{\tau_{cr}^*}{\tau_b^{*'}}\right)\left(1 - \boldsymbol{Fr}^2\right) \quad \textbf{(9)}$$

where

$\boldsymbol{Fr}$, Froude number, [-];

a , the constant coefficient used to calibrate the bed load transport capacity, [-].

b , the constant coefficient used to calibrate the bed load transport capacity, [-];

β , shape factor, [-];

$\tau_b^{*'} = \dfrac{\mu u_*^{\;2}}{\Delta g D}$, dimensionless grain shear stress;

$\tau_{cr}^* = \dfrac{u_{*cr}^{\;2}}{\Delta g D}$, dimensionless critical bed shear stress.

Based on a mathematical analysis for dune propagation, Fredsoe (1982) proposed the following dune height predictor:

$$\frac{H}{h} = \frac{2}{3}\left(1 - \frac{\tau_{cr}^*}{\tau_b^{*'}}\right)\left\{2 + \frac{1}{3}\left(1 - \frac{\tau_{cr}^*}{\tau_b^{*'}}\right)\right\}^{-1} \quad \textbf{(10)}$$

Based on the analysis of flume and field data, van Rijn (1984b, 1993) proposed a systemised relationship to predict bed form type and height. His classification system was based on the three dimensionless parameters $\boldsymbol{T}$, $\boldsymbol{Fr}$ and $\boldsymbol{D_*}$, where:

$\boldsymbol{D_*} = D_{50}\left(\dfrac{\Delta . g}{\nu^2}\right)^{\frac{1}{3}}$, particle diameter parameter, [-];

$\boldsymbol{T} = \dfrac{\tau_b^{'} - \tau_{cr}}{\tau_{cr}}$, bed shear stress parameter, [-];

ν = kinematic viscosity coefficient, [m^2/s].

When ripples are present, the height of the bed forms can be estimated from:

$$\frac{H}{h} = 0.02\left(1 - e^{-0.1\boldsymbol{T}}\right)\left(10 - \boldsymbol{T}\right) \quad \textbf{(11)}$$

When dunes are present, the height of the bed forms can be estimated from:

$$\frac{H}{h} = 0.11\left(\frac{D_{50}}{h}\right)^{0.3}\left(1 - e^{-0.5\left(\frac{\tau_b^{*'}}{\tau_{cr}^*} - 1\right)}\right)\left(26 - \frac{\tau_b^{*'}}{\tau_{cr}^*}\right) \quad \textbf{(12)}$$

The predicted dune height for the flume conditions of $h/D_{50} = 200$ and $\boldsymbol{Fr}$ = 0.4 (Ribberink, 1987) given by Equations **(9)**, **(10)** and **(11)** are compared in Figure 3. For these particular conditions, the van Rijn and Gill results are in reasonable agreement. However, the results in Figure 3 are not universal. As shown in the previous description, each predictor includes different variable dependencies. Therefore, different predictors may well produce very different results under some circumstances.

3 Vertical Exchange of Bed Material

The fractional source term S_j, is introduced to represent the vertical exchange of bed material. This term is introduced in the advection-dispersion equation for the suspended load transport, in the sediment mass balance equation and in the fractional sediment mass balance equation for bed material sorting. The change in bed material composition is quantified by evaluating the net influx or efflux of bed load and suspended load, which is represented by the term $\sum_{j=1}^{J_{frac}} S_j$, and any exchange with the sub-stratum materials.

Assuming all sediment size classes are homogeneously distributed throughout the active layer, i.e. no stratification of sediment sizes, at any time step the mass conservation of each sediment size class can be analysed as follows (Bennett and Nordin, 1977; Holly, 1988; Rahuel et al., 1989;

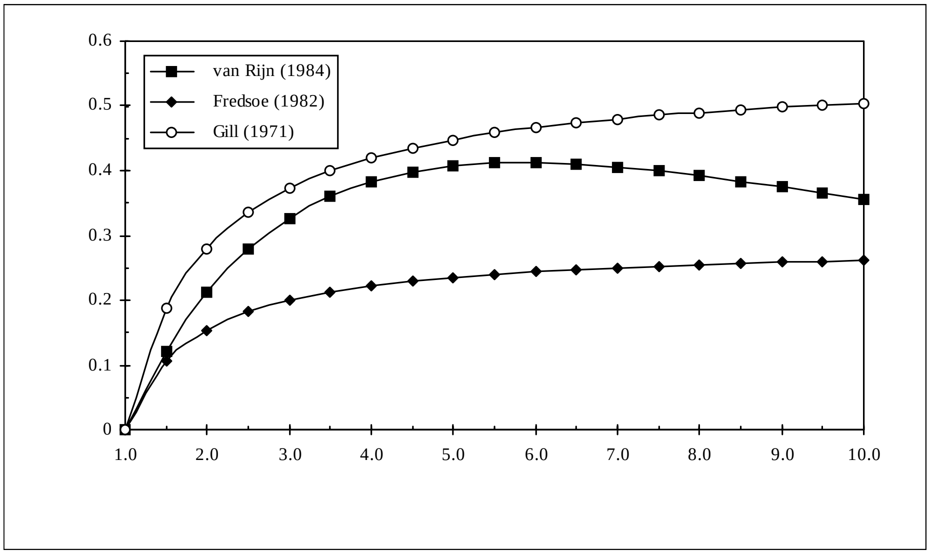

Figure 3 Comparison of dune heights given by various predictors with the flow conditions: $h/D_{50} = 200$ Fr = 0.4, β = 0.5 and b = 3 for MPM transport formula (adapted from Ribberink, 1987).

Holly and Rahuel, 1990):

$$\frac{\partial V_{active_j}}{\partial t} = Q_{sb_j} - \left(Q_{sb_j} + \frac{\partial Q_{sb_j}}{\partial x}\Delta x\right) - S_j.\Delta x - \frac{\partial V_{sub-stratum_j}}{\partial t} + q_{sb_{lat_j}}.\Delta x \qquad \textbf{(13)}$$

Referring to Figure 4, the following relations can be derived:

$$V_{active_j} = (1-p).f_j.B_{act}.\delta.\Delta x$$

$$V_{sub-stratum_j} = (1-p).\zeta_j.B_{act}.(z_b - \delta).\Delta x$$

To define ζ_j, three distinct conditions for the bed elevation changes over a time step Δt are considered (see Figure 5):

a) In the case of a net deposition or an aggrading bed level, part of the active layer material leaves the layer and becomes the sub-stratum or inactive layer over a time step Δt (Figure 5 (i)).
b) In the case of a net erosion, i.e. a degrading bed level, but the interface between the active layer and the sub-stratum layer is rising or resting, there is no contribution of the sub-stratum to the active layer over a time step Δt (Figure 5 (ii)).
c) In the case of a degrading bed level and the interface between the active layer and the sub-stratum layer is also descending, there is a contribution of the sub-stratum layer to the material and its gradation in the active layer over a time step Δt (Figure 5 (iii)).

Taking into account the three possible conditions, the term ζ_j can be formulated as follows (Rahuel, et al., 1989; Holly and Rahuel, 1990):

$$\zeta_j = \varepsilon_1 \left\{\varepsilon_2 . f_j + (1-\varepsilon_2).f_{1j}\right\}$$

where

ε_1 , a switch equal to 0 or 1 for river bed aggradation or degradation, respectively;

ε_2 , a switch equal to 0 or 1 for the interface between the active layer and the sub-stratum layer descending or rising, respectively.

f_{1j} , volume fraction of size class *j* being present in the sub-stratum layer, [-].

Incorporating the vertical exchange of the sediment, Equation **(3)** can be rewritten as (Holly and Rahuel, 1990):

$$(1-p)B_{act}\frac{\partial f_j\delta}{\partial t} + \frac{\partial Q_{sb_j}}{\partial x} + S_j + (1-p).\zeta_j.B_{act}\left(\frac{\partial z_b}{\partial t} - \frac{\partial \delta}{\partial t}\right) = q_{sb_{lat_j}} \qquad \textbf{(14)}$$

Equation **(14)** is used to analyse the changes in bed material composition.

Conservation of the mass of suspended load is evaluated by means of the advection-dispersion equation for suspended load transport:

$$\frac{\partial(\bar{c}_j A)}{\partial t} + \frac{\partial(\bar{c}_j Q)}{\partial x} = \frac{\partial}{\partial x}\left(AK\frac{\partial \bar{c}_j}{\partial x}\right) + S_j + \left(q_{lat}.\bar{c}_{lat_j}\right) \qquad \textbf{(15)}$$

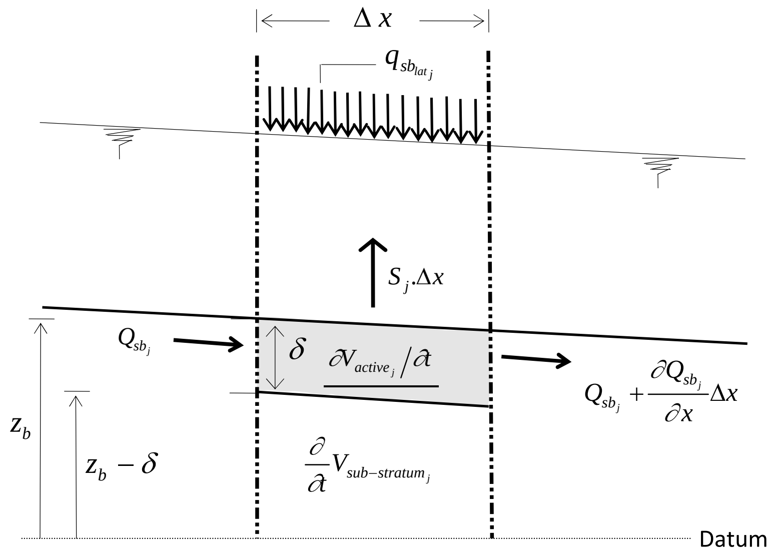

Figure 4 Schematised sediment mass conservation at the active layer.

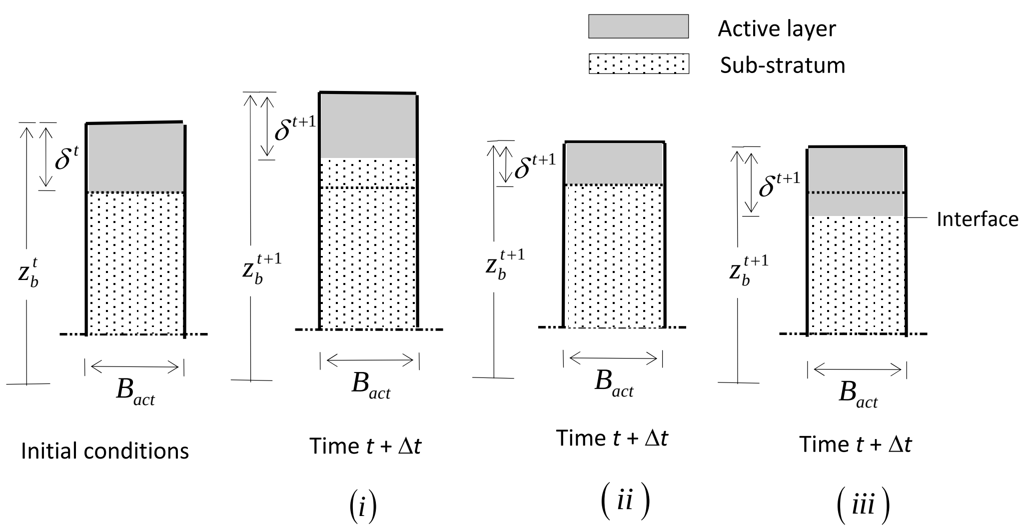

Figure 5 Schematised changes in bed level and the interface between the active layer and the sub-stratum layer.

The source term S_j needs to be further quantified. An instantaneous change in sediment transport capacity induces a concentration difference between the bed load and the suspended load zones across the line *B* - *B* in Figure 6. The concentration difference causes a flux from the higher to the lower concentration (Bennett and Nordin, 1977; van Rijn, 1984a; Celik and Rodi, 1988).

The source term for the j^{th} sediment size class S_j, is composed by two independent processes, deposition and entrainment. The net source term is:

$$S_j = -S_{d_j} + S_{e_j} \tag{16}$$

where

S_{d_j} , the deposition component or the downward flux of the j^{th} sediment size class due to gravity, [m³/s/m];

S_{e_j} , the entrainment component or the upward flux of the j^{th} sediment size class due to turbulence, [m³/s/m].

The rate of deposition of the j^{th} size class is calculated by considering the concentration at the lower edge of the suspended load layer c_{d_j} and the sediment settling velocity w_j, as follows:

$$S_{d_j} = B_{act}\ w_j\ c_{d_j} \quad \textbf{(17)}$$

Since the designed model is a one-dimensional model, it is necessary to obtain the value of c_{d_j} based on the cross-sectional averaged suspended load concentration. For this purpose, the following relation proposed by Bennet and Nordin (1977) is adopted:

$$c_{d_j} = \bar{c}\,\frac{hw_j}{\varepsilon_s}\left(1-\exp\left(-\frac{w_j}{\varepsilon_s}\left(h-\delta_b\right)\right)\right)^{-1} \quad \textbf{(18)}$$

where

δ_b , thickness of the bed load layer, [m];

ε_s , the vertical eddy diffusivity for the sediment, [m^2/s].

The above relation was derived based on integration of the depth concentration profile of the suspended load and averaging it over the depth. The value of ε_s was taken based on the depth-averaged value of a logarithmic velocity profile. Assuming the local bed slope to equal the friction slope, the following relation of ε_s was proposed (Graf, 1971; Bennett and Nordin, 1977; van Rijn, 1982):

$$\varepsilon_s = \frac{\kappa u h g^{0.5}}{6C}$$

where

κ , von Karman's coefficient, which has a clear water value of approximately 0.4, [-];

C , Chezy coefficient for bed roughness, [m$^{0.5}$/s].

Two possible channel bed conditions steer the formulation of the entrainment process:

a) The entrainment always occurs at its maximum rate S_{e_j} , for conditions where the bed consists of an unlimited amount of loose sediment material.
b) When there is a fixed bed and sediment is carried by the flow, all the deposited sediment will be re-entrained if the deposition rate is lower than the S_{e_j} .

The maximum entrainment component, S_{e_j} is taken as (Holly and Rahuel, 1990; Celik and Rodi, 1988):

$$S_{e_j} = B_{act}\, w_j\, f_j\, \alpha_j\, c_{e_j} \quad \textbf{(19)}$$

where

c_{e_j} , the near bed equilibrium concentration or reference concentration for sediment size class j, [m^3/m^3];

α_j , the ratio of suspended load to the total load transport (Rijn, 1984a; Laursen, 1958), [-].

In the present model, the reference concentration is defined to be equal to the bed load concentration at the upper edge of the bed load layer. The reference concentration for the j^{th} size class c_{e_j} , can be estimated from the following relation due to van Rijn, 1984a:

$$c_{e_j} = 0.015\,\frac{D_{50}}{\delta_b}\,\frac{\boldsymbol{T}^{1.5}}{\boldsymbol{D}_*} \quad \textbf{(20)}$$

where the thickness of the bed load layer δ_b, can be estimated as follows:

a) for channel with a flat bed δ_b = 2 to 10 D_{50},
b) when bed forms are present, $\delta_b = k_s$ (in which k_s is the effective bed roughness height) or δ_b = 0.5 H, moreover
c) van Rijn recommended that $\delta_b \geq 0.01\,h$.

The ratio of suspended load to the total load transport α, can be interpolated from the following Table 1 due to van Rijn, 1984a and Laursen, 1958.

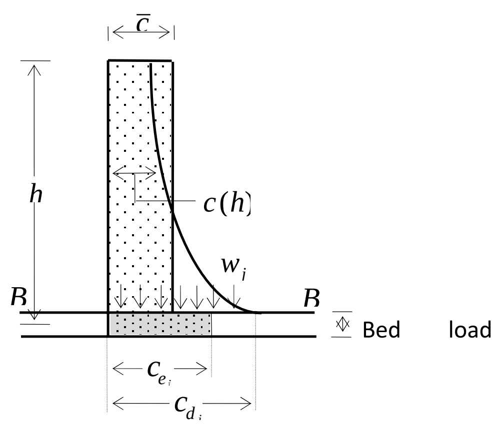

Figure 6 Schematised vertical exchange between suspended load and bed load.

Table 1 Ratio of the suspended load to the total load, α (adapted from van Rijn, 1984a and Laursen, 1958).

u_*/w_s	0.4	0.6	0.8	1.0	2.0	4.0	6.0	8.0	10.0
α	0	0.1	0.17	0.25	0.55	0.87	0.95	0.98	1.0

4 Alluvial Bed Roughness

In the case of a moveable bed consisting of non-cohesive sediments, the effective bed roughness height k_s, mainly consists of two components (van Rijn, 1984b and 1993):

a) the grain roughness $k_s^{'}$, generated by the skin friction forces, and
b) the form roughness $k_s^{''}$, generated by the pressure forces acting on the bed.

Simons and Richardson (1961) and van Rijn (1984b and 1993) have shown that the effective bed roughness for a given bed material size is not constant. It depends on the flow conditions, i.e. the depth, velocity and sediment transport rate. The difficulty is that the bed configuration and its roughness also influence these hydraulic variables. Hence, under continuously varying discharges, the bed form dimensions and the roughness coefficient are also varying with the flow conditions.

The effective bed roughness height, k_s can be related, for an example to the Chezy coefficient C through the Chezy equation as:

$$C = 18.0 \log\left(\frac{12R}{k_s}\right) \quad \textbf{(21)}$$

Equation **(21)** only applies to the hydraulically rough, turbulent flow regime.

The methods to estimate the effective bed roughness can basically be grouped into two main approaches (van Rijn, 1993):

a) methods based on bed form-related and grain-related parameters, such as bed form length, height and steepness and bed material size,
b) methods based on integral parameters, such as the mean depth, mean velocity and bed material size.

Since the characteristics of the bed forms are also required to determine the thickness of the active layer, the first approach will only be discussed in this paper.

The effective roughness height of Nikuradse k_s, is composed of a grain-related part and a form-related part:

$$k_s = k_s^{'} + k_s^{''} \quad \textbf{(22)}$$

where

$k_s^{'}$, grain roughness height, [m];

$k_s^{''}$, form roughness height, [m].

1) The grain roughness height, $k_s^{'}$.

The grain roughness is defined as the roughness of individual moving or non-moving sediment particles present in the top layer of a plane, moveable or non-moveable bed (van Rijn, 1982). Various researchers have conducted experiments to determine the grain roughness value of moveable and non-moveable beds (Gladki, 1975; Hey, 1979; van Rijn, 1982 and 1984b; Lyn, 1991). These researches were conducted by using mainly sand and gravel beds and can be summarised as follows:

a) The grain roughness in the lower transport regime is mainly related to the largest particles in the top layer of the bed. Gladki (1975) and Hey (1979) used D_{84} to characterise the grain roughness height, whereas van Rijn (1982, 1984b, 1993) and Lyn (1991) proposed D_{90} for that purpose.
b) the grain roughness height of a moveable plane bed seems to be larger than that of a rigid plane bed. This phenomenon is concluded by observing the resistance of a plane bed in a flume under a sediment-water mixture and clear water conditions (Lyn, 1991). The increment in the $k_s^{'}$ value is caused by the interaction of the flow with sediment particles in the near bed region. The collisions between the transported sediment particles and the channel bed reduce the fluid velocity. This results in an additional shear effect to the fluid shear. Therefore, the $k_s^{'}$ value increases.

Based on the available field and flume data for lower transport and upper transport regimes, van Rijn (1982) quantified the grain-related effective roughness as follows:

$k_s^{'} = 3D_{90}$ for $\theta < 1$ (lower transport regime) **(23.a)**

$k_s^{'} = 3\theta D_{90}$ for $\theta \geq 1$ (upper transport regime) **(23.b)**

where

$\theta = \dfrac{u_*^2}{\Delta g D_{50}}$, particle mobility parameter, [-];

$u_* = \frac{u\sqrt{g}}{C}$, bed shear velocity, [m/s];

$C = 18\log\left(\frac{12h}{k_s^{'}+3.3\frac{\nu_m}{u_*}}\right)$, Chezy roughness coefficient, [m$^{0.5}$/s];

$\nu_m = \frac{\eta_m}{\rho_m}$, kinematic viscosity of the sediment-water mixture, [m^2/s].

Equation **(23.b)** can only be solved by iteration and results in a conservative estimate for $k_s^{'}$ (van Rijn, 1993). The $k_s^{'}$ values from equation **(23)** and the data of Winterwerp et al. (1990) are in good agreement. In addition, van Rijn (1993) proposed a minimum value of $k_s^{'}$ = 0.01 m for flows in the upper transport regime with high concentrations over a plane bed.

2) The form roughness height, $k_s^{''}$.

The effective form roughness is related to the bed form height, the bed form steepness (H/λ) and the bed form shape (β). Since several types of bed forms may exist at the same time on an alluvial river bed, it is proposed to determine the overall form roughness by summing the contribution from each type of bed form as follows (van Rijn, 1993) :

$$k_s^{''} = k_{s,r}^{''} + k_{s,d}^{''} + k_{s,sw}^{''} \quad \textbf{(24)}$$

where

$k_s^{''}$, overall effective form roughness, [m];

$k_{s,r}^{''}$, form roughness related to ripples, [m];

$k_{s,d}^{''}$, form roughness related to dunes, [m];

$k_{s,sw}^{''}$, form roughness related to symmetrical sand waves, [m].

The form roughness height for ripples can be predicted by the following relationship (van Rijn, 1993 and 1984b):

$$k_{s,r}^{''} = 20\beta_r H_r \left(\frac{H_r}{\lambda_r}\right) \quad \textbf{(25)}$$

where

H_r , ripple height, evaluated by means of Equation **(11)**, [m];

λ_r , 0.5 h, ripple length, [m];

β_r , ripple presence factor (β_r = 1.0 for ripples alone; and β_r = 0.7 for ripples superimposed on dunes or sand waves).

The effective form roughness height for dunes can be predicted from the following relationship (van Rijn, 1993 and 1984b):

$$k_{s,d}^{''} = 1.1\,\beta_d\,H_d\left(1-e^{-\left(\frac{25\,H_d}{\lambda_d}\right)}\right) \quad \textbf{(26)}$$

where

H_d , dune height, predicted by equation **(12)**, [m];

λ_d = 7.3 h, dune length, [m];

β_d , form factor for dunes, [-].

The leeside slopes of the field dunes are much smaller than those of flume dunes. This condition results in a less important flow separation. To account for this, the dune form factor correction is applied. For field conditions β_d = 0.7 is proposed, whereas for flume condition β_d = 1.0 (van Rijn, 1984b and 1993).

Sand waves are defined as symmetrical bed forms with lengths much larger than the water depth. Since the leeside slopes of symmetrical sand waves are relatively mild, flow separation will not occur. Therefore, the form roughness of symmetrical sand waves is assumed to be zero (van Rijn, 1993).

4) Bed form adjustment due to unsteady flow conditions.

Field observations of bed forms show that they adjust with a time lag to changes in the discharge (Carey and Keller, 1957; Allen, 1976a and 1976b; Bayazit, 1969; Fredsoe, 1979 and 1981; Yalin, 1977). The observations were undertaken mainly on rivers where sand was the dominant bed material. Carey and Keller (1957) noted that sand dunes on the Mississippi River were smaller than predicted during the rising limb of a flood wave. During the falling limb however, their dimensions were larger than predicted.

Considering the contribution of bed forms to the effective alluvial bed roughness, it can be concluded that the time required by an alluvial reach to adapt to new flow conditions, also influences the alluvial bed roughness. A prediction of the alluvial bed roughness by any predictor under steady flow conditions will overestimate the roughness on the rising limb of the flood wave, and underestimate it on the falling stage. Therefore, the time scales for bed form adjustment should be reproduced in the model. The temporal lag model for an alluvial stream proposed by Phillips (1984) and Phillips and Sutherland (1990) is incorporated in the present model to reproduce the bed form adjustments due to changes in hydrodynamics. A detailed discussion of the temporal lag model is given by Moerwanto (2011).

METHODOLOGY

To achieve an efficient and robust sub-model that could easily communicate with the main model, the structure of sub-model for simulating selective transport and armoured layer development in non-uniform bed materials under unsteady flow conditions is designed by implementing the modularity advantages of finite element solution. The performances of the sub-model are steps by steps verified by comparing its prediction results with set of data resulted from physical hydraulic laboratory tests. The results of this verification are presented in the second part of this research paper.

SETTING UP OF SELECTIVE TRANSPORT AND ARMOURED LAYER DEVELOPMENT SUB-MODEL

To achieve the desirable model, the following field phenomena of bed load transport in non-uniform bed materials under unsteady flow conditions are incorporated and simulated by the selective transport and armoured layer development sub-model:

a) Formation of alluvial bed roughness
b) Prediction of active layer thickness,
c) Schematization of bed materials exchange,
d) Evaluation of actual size fraction sediment transport, capacity under unsteady flows conditions by including the temporal and spatial lag effects of the alluvial reach, and
e) Development of armoured layer

The structure diagram of the designed selective transport and armoured layer development sub-model as a subroutine of the being developed A Riverine Fully Coupled Finite Element Model with Sediment Transport Sub-processes is shown in Figure 6.

1) Adopted method for formulating size fraction transport of non-uniform bed materials

Many of the transport formulae can be recast into the form of a proportional relation based on the excess of the grain shear stress beyond that needed for incipient motion, $\left(\tau_b^{'} - \tau_{cr}\right)$. Therefore, it can be inferred that to cope with the non-uniform bed material both formulations of the exposure correction factor, i.e. modifying the grain shear stress $\tau_b^{'}$, or adjusting the critical bed shear stress τ_{cr}, should yield a similar result. The following techniques are adopted in the bed load sub-model of the present model to cope with the non-uniform bed material:

a) in cases where the modification of a particular transport formula has been proposed, the modified version of the transport formula is adopted (viz. Ackers and White, MPM and Paintal formulae), otherwise
b) the exposure correction factor for the critical bed shear stress is incorporated into a sediment transport formula derived for uniform bed material.

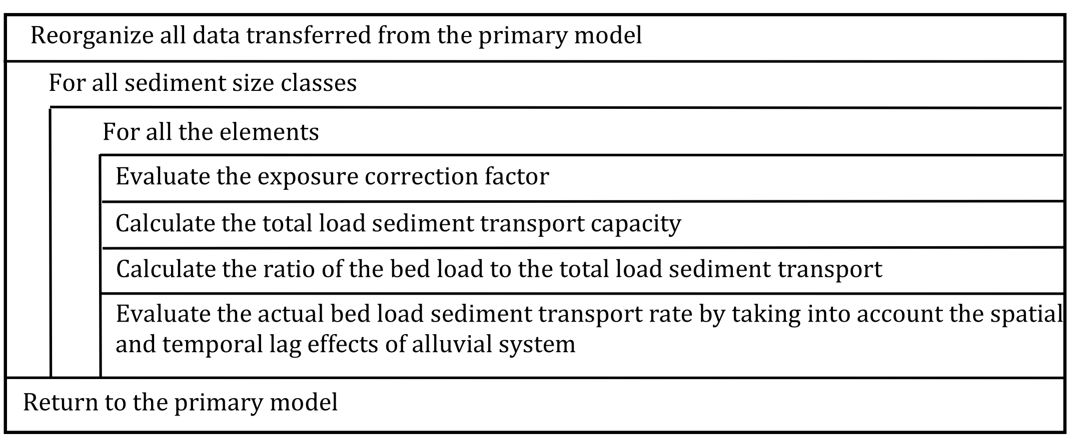

Figure 6 Structure diagram of the bed load sediment transport sub-model.

2) Adopted predictor for the active layer thickness

Bearing in mind that the present model is developed with a modular structure, any predictor can be easily incorporated and used as a tuned variable. Since there is no outstanding method, the following considerations can guide the selection of an appropriate predictor for the active layer thickness:

a) Since the bed load layer is a part of the active layer, the active layer thickness must be thicker than the bed load layer.
b) Unless extensive data for calibration is available, a constant active layer thickness as proposed by Bennett and Nordin (1977) is preferable. Even though it does not represent the active layer development, its simplicity compared to other methods is an attraction in itself. This method narrows down the number of parameters in the bed material sorting sub-process. A relationship based on bed form height can be used to estimate a reasonable thickness for the active layer.
c) Due to the range of flume and field data used to develop the van Rijn bed form predictor (1984b, 1993), this predictor is preferred to estimate the active layer thickness.

3) Adopted alluvial bed roughness predictor

The dynamic wave approach for the momentum equation is implemented in the present model. As a consequence, the temporal lag effect on the effective alluvial bed roughness must also be included. To satisfy these requirements, the following features are employed in the present model:

a) The Chezy coefficient is used to represent the alluvial bed roughness. Hence, the predicted bed roughness given by any predictor must first be translated into a Chezy coefficient before being incorporated in the discrete momentum equation.
b) Unless an alluvial bed roughness predictor which copes with unsteady flow conditions is developed, predictors which are based on the grain and bed form parameters are preferred. The contribution of grain roughness is basically independent of the flow unsteadiness. Meanwhile, the contribution of the bed forms depends on the unsteadiness of the flow. The clear separation in quantifying the contribution of the grain and the bed form roughness means that the required adjustments due to unsteady flow conditions are easier to be carried out. It also makes further verification and calibration easier.

The equivalent steady discharge model proposed by Phillips (1984) and Phillips and Sutherland (1990) is adopted to cope with thetemporal lag effect of the alluvial stream due to unsteady flow conditions. In Phillip's model, the unsteady flow hydrograph is basically transformed into a series of equivalent steady discharges. The hydrodynamic data determined from these steady discharges can be used to determine the geometric dimensions of the bed forms in unsteady flow conditions Moerwanto (2011).

CONCLUSIONS

The present study has resulted in a complete theoretical basis required to setup sub-model for simulating selective transport and armoured layer development in non-uniform bed materials under unsteady flow conditions.

To cope with non-uniform bed material, the exposure correction factor for the critical bed shear stress is incorporated in combination with a sediment transport formula derived for uniform bed material. However, in cases where the modification of a particular transport formula to suit the non-uniform bed material has been proposed, the modified version of the transport formula could also be adopted.

To evaluate the spatial and temporal variations of bed material composition, the thickness of the active layer needs to be determined. Since the bed load layer is a part of the active layer, the active layer thickness must be larger than the thickness of the bed load layer. The active layer thickness predictors which are based on the gradation of the bed material as proposed by Borah et al., (1982) or a constant thickness as proposed by Bennett and Nordin (1977) are preferable. In cases where a constant active layer thickness is chosen, the active layer thickness can be estimated from a relation based on the height of the bed forms.

The vertical exchange between the bed load sediment, the suspended load sediment and the bed material is represented by means of a source term. The source term is composed of two independent processes:

a) the deposition process which is evaluated by considering the concentration at the base of the suspended load layer and the sediment particle settling velocity, and
b) the entrainment process which is characterised by the sediment particle settling velocity, the reference concentration at the top of the bed load layer, the volume fraction of each size class present in the active layer and

the ratio of suspended load to the total load transport.

Since the dynamic wave approach is adopted in the designed model, the temporal lag effect on the alluvial bed roughness has to be included.

REFERENCES

Ackers, P. and W.R. White (1980). Bed material transport: A theory for total load and its verification. International Symposium on River Sedimentation, Beijing, Paper B10.

Allen, J.R.L. (1965). Sedimentation to the lee of small underwater sand waves: an experimental study. Journal of Geology, Vol. 73, pp. 95-110.

Ashida, K. and M. Michiue (1973). Studies on bed load transport rate in open channel flows. Proceedings of IAHR, International Symposium on River Mechanics. A.I.T., Bangkok, Vol. 1, Paper A-36, pp. 407-418.

Bayazit, M (1969). Resistance to reversing flow over moveable beds. Journal of the Hydraulics Division, ASCE, Vol. 95, No. HY4, pp. 1109-1127.

Bennett, J.P. and C.F. Nordin (1977). Simulation of sediment transport and armouring. Hydrological Sciences Bulletin, Vol. XXII, No. 4, pp. 555-569.

Borah, D.K., C.V. Alonso, and S.N. Prasad (1982). Routing graded sediments in streams: Formulations. Journal of Hydraulic Engineering, ASCE, Vol. 108, No. HY12, pp. 1486-1503.

Carey, W.C. and M.D. Keller (1957). Systematic changes in the beds of alluvial rivers, Journal of the Hydraulics Division, ASCE, Vol. 83, No. HY4, 24 p.

Celik, I. and W. Rodi (1988). Modeling suspended sediment transport in non-equilibrium situations. Journal of Hydraulic Engineering, ASCE, Vol. 114, No.10, pp. 1157-1191.

Chin, C.O. (1985). Stream bed armouring. Report no. 403 - Department of Civil Engineering, University of Auckland.

Chin, C.O., B.W. Melville and A.J. Raudkivi (1994). Streambed armouring. Journal of Hydraulic Engineering, ASCE, Vol 120, No. 8, pp. 899-918.

Day, T.A.J. (1980). A study of the transport of graded sediment. Hydraulic Research Station Report No. INT 190, Wallingford.

Egiazaroff, I.V. (1965). Calculation of non-uniform sediment concentration. Journal of the Hydraulics Division, ASCE, Vol. 91, No. HY4, pp. 225-247.

Einstein, H.A. (1950). The bed-load function for sediment transportation in open channel flow. Technical Bulletin No. 1026, U. S. Department of Agriculture, Washington, D. C.

Fredsoe, J. (1979). Unsteady flow in straight alluvial streams, Part 2: Modification of individual dunes. Journal of Fluid Mechanics, Vol. 91, pp. 479-512.

Fredsoe, J. (1981). Unsteady flow in alluvial streams, Part 3: Transition from dunes to plane bed. Journal of Fluid Mechanics, Vol. 102, pp. 431-453.

Fredsoe, J. (1982). Shape and dimensions of stationary dunes in rivers. Journal of the Hydraulics Division, ASCE, Vol. 108, No. HY8, pp. 932-947.

Gessler, J. (1967). The beginning of bed load movement of mixtures investigated as natural armouring in channels. W.M. Keck Laboratory of Hydraulic and Water resources, California Institute of Technology, California.

Gill, M.A. (1971). Height of sand dunes in open channel flows. Journal of the Hydraulics Division, ASCE, Vol. 97, No. HY12, pp. 2067-2074.

Gladki, H. (1975). Discussions of determination of sand roughness for fixed beds. Journal of Hydraulic Research, IAHR, Vol. 13, No. 2.

Graf, Walter H. (1971). Hydraulics of sediment transport. McGraw-Hill, New York.

Hey, R.D. (1979). Flow resistance in gravel-bed rivers. Journal of the Hydraulics Division, ASCE, Vol. 105, No. HY4, pp. 365-379.

Holly, F.M. Jr. (1988). CHARIMA and SEDICOUP codes for riverine mobile-bed simulation. Second Seminar on Stream Sedimentation Models, Sedimentation Sub-committee, Interagency Advisory Committee on Water Data, pp. 263 - 351, Denver.

Holly, F.M. Jr. and J.L. Rahuel (1990). New numerical/physical framework for mobile bed modeling, Part I Numerical and physical principles. Journal of Hydraulic Research, IAHR, Vol. 28, No. 4, pp. 401- 416.

Laursen, E.M. (1958). The total sediment load of streams. Journal of the Hydraulics Division, ASCE, Vol. 84, No. HY1, pp. 1530-1 - 1530-36.

Lyn, D.A. (1991). Resistance in flat bed sediment laden flows. Journal of Hydraulic Engineering, ASCE, Vol. 117, No. 1, pp. 94-113.

Moerwanto, A.S. (2011). Temporal Lag Effects of Alluvial System Under Unsteady Flow Conditions, Jurnal Teknik Hidraulik, Vol. 2, no. 1, Juni 2011, pp. 1-16.

Phillips, B.C. (1984). Spatial and temporal lag effects in bed-load sediment transport - Civil Engineering Research Report No. 84-10 - Reprint of PhD Thesis, Department of Civil Engineering, University of Canterbury.

Phillips, B.C. and A. J. Sutherland (1990). Temporal lag effects in bed load sediment transport. Journal of Hydraulic Research, IAHR, Vol. 28, No. 1, pp. 5-23.

Proffitt, G.T. and A.J. Sutherland (1983). Transport of non-uniform sediment. Journal of Hydraulic Research, IAHR, Vol. 21, No. 1, pp. 33-43.

Rahuel, J.L., F. M. Holly, J. P. Chollet, P. J. Belleudy and G. Yang (1989). Modeling of river bed evolution for bed load sediment mixture. Journal of Hydraulic Engineering, ASCE, Vol. 115, No. 11, pp. 1521-1542.

Ribberink, J.S. (1987). Mathematical modeling of one-dimensional morphological changes in rivers with non-uniform sediment. Delft University of Technology, PhD Thesis.

Rijn, L.C. van (1982). Equivalent roughness of alluvial bed. Journal of the Hydraulics Division, ASCE, Vol. 108, No. HY10, pp. 1215-1218.

Rijn, L.C. van (1984a). Sediment transport, Part II Suspended load transport. Journal of Hydraulic Engineering, ASCE, Vol. 110, No. 11, pp. 1613-1641.

Rijn, L.C. van (1984b). Sediment transport, Part III Bed forms and alluvial roughness. Journal of Hydraulic Engineering, ASCE, Vol. 110, No. 2, pp. 1733-1754.

Rijn, L.C. van (1993). Principles of sediment transport in rivers, estuaries and coastal seas. Aqua Publications, Amsterdam.

Simons, D.B. and E.V. Richardson (1961). Forms of bed roughness in alluvial channels. Journal of Hydraulic Engineering, ASCE, Vol. 87, No. HY3, pp. 87-105.

Yalin, M.S. (1977). On the physical modeling of dunes. Proceedings of XVIIth Congress, IAHR, Baden-Baden, Vol. 1, Paper A4, pp. 25-32.

HALAMAN INI SENGAJA DIKOSONGKAN